The massive photon in pseudoclassical mechanics

1983 J. Phys. A: Math. Gen. 16 1993

(http://iopscience.iop.org/0305-4470/16/9/023)

J. Phys. A: Math. Gen. 16 (1983) 1993–1998. Printed in Great Britain

The massive photon in pseudoclassical mechanics

A Barducci† and L Lusanna‡

† Istituto di Fisica Teorica dell'Università di Firenze and Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, 50125 Firenze, Italy
‡ Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, 50125 Firenze, Italy

Received 22 November 1982

Abstract. A set of constraints for a pseudoclassical particle is given, which after quantisation reproduces the wave equation associated with the Stückelberg Lagrangian. By path-integral quantisation the covariant propagators of the photon in QED are recovered.

In a previous paper (Barducci and Lusanna 1982) we gave a description of a massless particle belonging to the $D(1,0)\oplus D(0,1)$ representation of SL$(2, C)$ by means of pseudoclassical mechanics. Two complex Grassmann four-vectors θ^μ, $\theta^{*\mu}$ were introduced to describe the helicity degrees of freedom and a suitable set of first-class constraints was introduced to describe the photon. By canonical quantisation with the Gupta–Bleuler prescription we obtained the one-photon wavefunctions in the Lorentz gauge, whereas by path-integral quantisation the non-covariant transverse propagator was found. The breaking of manifest covariance is due to the presence of the light-like transverse polarisation vectors.

In this paper we propose a different set of first-class constraints, which, after canonical quantisation, reproduce the Proca-like wave equation (Itzykson and Zuber 1980)

$$(\Box+\mu^2)A^\mu(x)-(1-\lambda)\partial^\mu\partial_\nu A^\nu(x)=0 \tag{1}$$

which results from the Stückelberg Lagrangian (Stückelberg 1938)

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\mu^2A_\mu A^\mu-\tfrac{1}{2}\lambda\,(\partial_\mu A^\mu)^2. \tag{2}$$

This Lagrangian for $\mu\neq 0$ and $\lambda\to 0$ gives the Proca theory for massive vector fields, while for $\lambda\neq 0$, $\mu\to 0$ it is used for the quantisation of the electromagnetic field in the covariant gauge (Itzykson and Zuber 1980, Feldman and Matthews 1962).

Let us introduce the two constraints

$$\chi=P^2-\mu^2+(1-\lambda)P\cdot\theta^*P\cdot\theta\approx 0, \qquad \phi=\theta^*\cdot\theta\approx 0, \tag{3}$$

where P^μ is the canonical momentum of the particle with position x^μ, μ its mass and θ^μ, $\theta^{*\mu}$ are Grassmann four-vectors. These variables satisfy the Poisson brackets (Barducci and Lusanna 1982)

$$\begin{aligned}&\{x^\mu, P^\nu\}=-\eta^{\mu\nu},\\ &\{\theta^\mu,\theta^{*\nu}\}=\mathrm{i}\eta^{\mu\nu},\end{aligned}\qquad \eta=(+,-,-,-), \tag{4}$$

and the two constraints turn out to be first class.

The Dirac Hamiltonian is

$$H_{\rm D}=\alpha(\tau)\chi+\beta(\tau)\phi \tag{5}$$

where $\alpha(\tau)$, $\beta(\tau)$ are the arbitrary Lagrange multipliers.

The Hamiltonian equations associated with $H_{\rm D}$ are

$$\begin{aligned} &\dot{x}^{\mu}=\{x^{\mu},H_{\rm D}\}\approx-2\alpha[\eta^{\mu\nu}+\tfrac{1}{2}(1-\lambda)(\theta^{*\mu}\theta^{\nu}+\theta^{*\nu}\theta^{\mu})]P_{\nu},\\ &\dot{P}^{\mu}=0,\qquad \dot{\theta}^{\mu}\approx\mathrm{i}G^{\mu\nu}(P,\tau)\theta_{\nu},\qquad \dot{\theta}^{*\mu}\approx-\mathrm{i}G^{\mu\nu}(P,\tau)\theta^{*}_{\nu},\end{aligned} \tag{6}$$

where $G^{\mu\nu}(P,\tau)=\beta(\tau)\eta^{\mu\nu}+\alpha(\tau)(1-\lambda)P^{\mu}P^{\nu}$. The last two equations have the following solution which will be needed later:

$$\begin{aligned} &\theta^{\mu}(\tau)=\left(\exp\left(\mathrm{i}\int_{\tau_{\rm i}}^{\tau}\mathrm{d}\tau'\,G(P,\tau')\right)\right)^{\mu\nu}\theta_{\mathrm{i}\nu},\\ &\theta^{*\mu}(\tau)=\left(\exp\left(\mathrm{i}\int_{\tau}^{\tau_{\rm f}}\mathrm{d}\tau'\,G(P,\tau')\right)\right)^{\mu\nu}\theta^{*}_{\mathrm{f}\nu}\end{aligned} \tag{7}$$

where $\theta^{\mu}_{\rm i}$ and $\theta^{*\mu}_{\rm f}$ are the boundary conditions.

The constraints (3) could have been deduced from the Lagrangian

$$L=-\tfrac{1}{2}\mathrm{i}(\theta^{*}\cdot\dot{\theta}-\dot{\theta}^{*}\cdot\theta)+\alpha\mu^{2}-\beta\theta^{*}\cdot\theta+\frac{1}{4\alpha}\left(\dot{x}^{2}-\frac{1-\lambda}{1-\frac{1}{2}(1-\lambda)\theta^{*}\cdot\theta}\dot{x}\cdot\theta^{*}\dot{x}\cdot\theta\right) \tag{8}$$

with $\alpha(\tau)$, $\beta(\tau)$ interpreted as einbeins (Barducci and Lusanna 1982). For $\mu\neq0$, α can be evaluated from (3) and (6), and (8) becomes

$$L=-\tfrac{1}{2}\mathrm{i}(\theta^{*}\cdot\dot{\theta}-\dot{\theta}^{*}\cdot\theta)-\mu\left(\dot{x}^{2}-\frac{1-\lambda}{1-\frac{1}{2}(1-\lambda)\theta^{*}\cdot\theta}\dot{x}\cdot\theta^{*}\dot{x}\cdot\theta\right)^{1/2}-\beta\theta^{*}\cdot\theta. \tag{9}$$

However, we cannot solve equations (6) for β, since it is multiplied by an odd factor.

The canonical quantisation is done by sending the Poisson brackets (4) into the commutation (antocommutation) relations ($\theta^{\mu}\to b^{\mu}$, $\theta^{*\mu}\to b^{+\mu}$)

$$\begin{aligned} &[\hat{x}^{\mu},\hat{P}^{\nu}]_{-}=-\mathrm{i}\eta^{\mu\nu},\qquad [b^{\mu},b^{+\nu}]_{+}=-\eta^{\mu\nu},\\ &[b^{\mu},b^{\nu}]_{+}=[b^{+\mu},b^{+\nu}]_{+}=0.\end{aligned} \tag{10}$$

The quantisation of both the constraints (3) presents ordering problems. For instance, if $\theta^{*\mu}\theta^{\nu}\to\alpha b^{+\mu}b^{\nu}+\beta b^{\nu}b^{+\mu}$ we have $\phi\to(\alpha-\beta)b^{+}\cdot b-4\beta$ and $\chi\to[1-\beta(1-\lambda)]\hat{P}^{2}-\mu^{2}+(1-\lambda)(\alpha-\beta)\hat{P}\cdot b^{+}\hat{P}\cdot b$. Since, as shown in equations (18)–(19), the known result for the propagator is obtained with $\phi\to\hat{\phi}=b^{+}\cdot b+1$ and $\chi\to\hat{\chi}=\hat{P}^{2}-\mu^{2}+(1-\lambda)\hat{P}\cdot b^{+}P\cdot b$ we assume this form of the quantum constraints. It must be noted that there is no choice of α and β which reproduces this form: if $\phi\to b^{+}\cdot b+1$ the coefficient of $\hat{P}^{2}$ cannot be one. Indeed, as shown in equation (20), the classical action associated with these quantum constraints is $S_{\rm cl}-\int_{\tau_{\rm i}}^{\tau_{\rm f}}\mathrm{d}\tau\,\beta(\tau)$, with $S_{\rm cl}$ given by (21). This would imply the use of the classical constraint $\phi'=\theta^{*}\cdot\theta+1\approx0$. But this constraint is inconsistent because $0=(\theta^{*}\cdot\theta)^{5}\not\approx1$. Therefore the path-integral quantisation of the previous classical model must be implemented with a measure modified to take into account the term $-\int_{\tau_{\rm i}}^{\tau_{\rm f}}\mathrm{d}\tau\,\beta(\tau)$ as in (20), if we assume as a normally ordered quantum Hamiltonian $\hat{H}_{\rm D}=\alpha\hat{\chi}+\beta\hat{\phi}$. This is an example of a pure quantum effect which has no classical counterpart. Indeed, reintroducing the right dimensions we would have $\phi'=\theta^{*}\cdot\theta+\hbar\approx0$ as $[\theta]=[\sqrt{\hbar}]$. As $\hbar$ has no classical limit, we obtain the classical constraint $\phi=\theta^{*}\cdot\theta\approx0$.

With the chosen quantum Dirac Hamiltonian $\hat{H}_{\mathrm{D}}$, the Schrödinger equation is

$$\mathrm{i}(\partial/\partial\tau)|\psi\rangle = \hat{H}_{\mathrm{D}}|\psi\rangle. \tag{11}$$

As in Barducci and Lusanna (1982), we restrict ourselves to the gauges $\dot{\alpha} = \dot{\beta} = 0$, $\alpha < 0$, in which the solution of (11) is

$$|\psi, \alpha\tau, \beta\tau\rangle = \exp[-\mathrm{i}(\alpha\hat{\chi} + \beta\hat{\phi})\tau]|\psi, 0, 0\rangle. \tag{12}$$

By defining $\tilde{\alpha} = \alpha\tau$, $\tilde{\beta} = \beta\tau$, we get a two-times formalism due to the arbitrariness of the constraints α and β. The physical states are then selected as the generalised solutions (12) which satisfy

$$[\hat{P}^2 - \mu^2 + (1-\lambda)P\cdot b^+ P\cdot b]|\psi\rangle_{\mathrm{PH}} = 0, \qquad (b^+\cdot b + 1)|\psi\rangle_{\mathrm{PH}} = 0. \tag{13}$$

They turn out to be (Barducci and Lusanna 1982)

$$|\psi\rangle_{\mathrm{PH}} = \int_0^{2\pi} \frac{\mathrm{d}\tilde{\beta}}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}\tilde{\alpha}\ |\psi, \tilde{\alpha}, \tilde{\beta}\rangle = \int_0^{2\pi} \frac{\mathrm{d}\tilde{\beta}}{2\pi} \exp[-\mathrm{i}\tilde{\beta}(b^+\cdot b + 1)] \int_{-\infty}^{\infty} \mathrm{d}\tilde{\alpha}\ |\psi, \tilde{\alpha}, 0\rangle. \tag{14}$$

The spinorial part of these states belongs to the subspace $b^{+\mu}|0\rangle$ of the 16-dimensional Hilbert space $|0\rangle$, $b^{+\mu}|0\rangle$, $b^{+\mu}b^{+\nu}|0\rangle \ldots$ due to $\hat{\phi} \approx 0$, which selects the states with occupation number one. If $|P\rangle$ are the eigenstates of the four-momentum operator $\hat{P}^\mu$, and if we define the following transverse and longitudinal states for $P^2 \neq 0$

$$|\mu\rangle_{\mathrm{T}} = (\eta^{\mu\nu} - P^\mu P^\nu/P^2)b_\nu^+|0\rangle, \qquad |\mu\rangle_{\mathrm{L}} = (P^\mu P^\nu/P^2)b_\nu^+|0\rangle, \tag{15}$$

then the physical states solutions of the first of equations (13) are

$$|P; P^2 = \mu^2/\lambda\rangle \otimes |\mu\rangle_{\mathrm{L}}, \qquad |P; P^2 = \mu^2\rangle \otimes |\mu\rangle_{\mathrm{T}}. \tag{16}$$

A general physical state will be

$$|\psi\rangle_{\mathrm{PH}} = \int_{P^2=\mu^2} \frac{\mathrm{d}^3\boldsymbol{P}}{2P^0} f(\boldsymbol{P})\, \mathrm{e}^{-\mathrm{i}P\cdot x}|\mu\rangle_{\mathrm{T}} + \int_{P^2=\mu^2/\lambda} \frac{\mathrm{d}^3\boldsymbol{P}}{2P^0} g(\boldsymbol{P})\, \mathrm{e}^{-\mathrm{i}P\cdot x}|\mu\rangle_{\mathrm{L}}. \tag{17}$$

In the four-momentum representation, the physical kernel between states annihilated by $(b^+\cdot b + 1)$ is (Schwinger 1951)

$$\begin{aligned} \hat{K}_{\mathrm{PH}}(P', P) &= \delta^4(P'-P) \int_{-\infty}^{+\infty} \mathrm{d}\tilde{\alpha}\ \exp\{-\mathrm{i}\tilde{\alpha}[P^2 - \mu^2 + (1-\lambda)P\cdot b^+ P\cdot b + \mathrm{i}\varepsilon]\} \\ &= \frac{\mathrm{i}\delta^4(P'-P)}{P^2 - \mu^2 + (1-\lambda)P\cdot b^+ P\cdot b + \mathrm{i}\varepsilon}. \end{aligned} \tag{18}$$

Its matrix elements between states $\langle 0|\varepsilon_A\cdot b$ and $\varepsilon_B\cdot b^+|0\rangle$, where ε_A^μ form an arbitrary set of real time-like polarisation vectors, $\varepsilon_A^\mu \eta_{\mu\nu} \varepsilon_B^\nu = \eta_{AB}$, are

$$\begin{aligned} (K_{\mathrm{PH}})_{AB} &= \varepsilon_A^\mu \varepsilon_B^\nu (K_{\mathrm{PH}})_{\mu\nu} = -\varepsilon_A^\mu \varepsilon_B^\nu \langle 0|b_\mu \hat{K}_{\mathrm{PH}} b_\nu^+|0\rangle, \\ (K_{\mathrm{PH}})_{\mu\nu} &= \mathrm{i}\delta^4(P'-P)\left(\frac{\eta_{\mu\nu}}{P^2-\mu^2+\mathrm{i}\varepsilon} + \frac{1-\lambda}{\lambda}\frac{P_\mu P_\nu}{(P^2-\mu^2+\mathrm{i}\varepsilon)(P^2-\mu^2/\lambda+\mathrm{i}\varepsilon)}\right) \\ &= \mathrm{i}\delta^4(P'-P)\left(\frac{\eta_{\mu\nu} - P_\mu P_\nu/\mu^2}{P^2-\mu^2+\mathrm{i}\varepsilon} + \frac{P_\mu P_\nu/\mu^2}{P^2-\mu^2/\lambda+\mathrm{i}\varepsilon}\right) \end{aligned} \tag{19}$$

as expected from (1).

To recover this result by means of the path integral, we follow the same steps as in Barducci and Lusanna (1982), starting from the evolution operator $U(\tau_f, \tau_i) = \exp[-i\hat{H}_D(\tau_f - \tau_i)]$, $\tau_f > \tau_i$. As there, the physical kernel turns out to be

$$K_{\rm PH}^{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(x_f, x_i) = \int_{-\infty}^{0} d\tilde{\alpha} \int_0^{2\pi} \frac{d\tilde{\beta}}{2\pi} K^{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(x_f, x_i),$$

$$\begin{aligned} K^{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(x_f, x_i) &= \langle x_f; \mu_1 \ldots \mu_n; \tau_f | x_i; \nu_1 \ldots \nu_n; \tau_i\rangle \\ &= \int d\mu_f\, d\mu_i\, \langle \mu_1 \ldots \mu_n | \theta_f\rangle\langle \theta_i^* | \nu_1 \ldots \nu_n\rangle \\ &\quad \times \langle x_f, \theta_f^* | \exp[-i\hat{H}_D(\tau_f - \tau_i)] | x_i, \theta_i\rangle \\ &= e^{-i\tilde{\beta}} \int d\mu_f\, d\mu_i\, \langle \mu_1 \ldots \mu_n | \theta_f\rangle\langle \theta_i^* | \nu_1 \ldots \nu_n\rangle \\ &\quad \times \int_{x_i,\theta_i,\tau_i}^{x_f,\theta_f^*,\tau_f} \mathcal{D}\left(x, \frac{P}{2\pi}\right) \mathcal{D}(\theta, \theta^*) \exp[-\tfrac{1}{2}\theta_f^* \cdot \theta(\tau_f) + iS_{\rm cl} - \tfrac{1}{2}\theta^*(\tau_i) \cdot \theta_i]. \end{aligned} \tag{20}$$

Here

$$\tilde{\alpha} = \alpha(\tau_f - \tau_i), \qquad \tilde{\beta} = \beta(\tau_f - \tau_i), \qquad d\mu = -\exp(\theta^* \cdot \theta)\, d^4\theta\, d^4\theta^*,$$

$$\mathcal{D}(\theta, \theta^*) = \lim_{N\to\infty} \prod_{k=1}^{N} d^4\theta_k\, d^4\theta_k^*$$

$\langle \mu_1 \ldots \mu_n | \theta_f\rangle$, $\langle \theta_i^* | \nu_1 \ldots \nu_n\rangle$, are the transition functions from the occupation number basis to the Grassmann coherent states and $S_{\rm cl}$ is the phase-space action

$$S_{\rm cl} \int_{\tau_i}^{\tau_f} d\tau\, \{-\tfrac{1}{2}i(\theta^* \cdot \dot{\theta} - \dot{\theta}^* \cdot \theta) - P \cdot \dot{x} - \alpha[P^2 - \mu^2 + (1-\lambda)P \cdot \theta^* P \cdot \theta] - \beta\theta^* \cdot \theta\}. \tag{21}$$

Let us note in (20) the term $e^{-i\tilde{\beta}}$ which is the previously discussed modification of the measure.

The integration over the Grassmann variables is Gaussian and is done by the shift to the classical solutions given by (17), with the result

$$\begin{aligned} &K^{\mu_1\ldots\mu_n;\nu_1\ldots\nu_n}(x_f, x_i) \\ &\quad = \exp(-i\tilde{\beta}) \int d\mu_f\, d\mu_i\, \langle \mu_1 \ldots \mu_n | \theta_f\rangle\langle \theta_i^* | \nu_1 \ldots \nu_n\rangle \\ &\quad \times \int_{x_i,\tau_i}^{x_f,\tau_f} \mathcal{D}\left(x, \frac{P}{2\pi}\right) \exp\left\{-\theta_{f\alpha}^* \left(\exp\left(i \int_{\tau_i}^{\tau_f} d\tau\, \alpha(1-\lambda)PP\right)\right)^{\alpha\beta} \theta_{i\beta}\, e^{i\tilde{\beta}} \right. \\ &\quad \left. - i \int_{\tau_i}^{\tau_f} d\tau [P \cdot \dot{x} + \alpha(P^2 - \mu^2)]\right\}. \end{aligned} \tag{22}$$

The remaining normalisation constant $N(\tau_f - \tau_i)$ after the shift to the classical solution turns out to be 1, by evaluating it with the lattice definition (Faddeev 1976) corresponding to a normal ordering. This path integral is not Gaussian, but actually we do not need its full expression as we are to project in $\tilde{\beta}$. In (20) the integration over $\tilde{\beta}$ can be done and a non-zero result is obtained only for $n=1$. It is

$$K^{\mu\nu}_{\mathrm{PH}}(x_f, x_i) = \int_{-\infty}^{0} \mathrm{d}\tilde{\alpha} \int \mathrm{d}\mu_f\, \mathrm{d}\mu_i \langle \mu|\theta_f\rangle\langle\theta_i^*|\nu\rangle \times \int_{x_i,\tau_i}^{x_f,\tau_f} \mathscr{D}\left(x, \frac{P}{2\pi}\right)\left\{-\theta^*_{f\alpha}\left(\exp\left(\mathrm{i}\alpha(1-\lambda)\int_{\tau_i}^{\tau_f} \mathrm{d}\tau\, PP\right)\right)^{\alpha\beta}\theta_{i\beta} \times \exp\left(-\mathrm{i}\int_{\tau_i}^{\tau_f} \mathrm{d}\tau[P\cdot\dot{x}+\alpha(P^2-\mu^2)]\right)\right\}. \tag{23}$$

As after (18), instead of between the states $|\mu\rangle = b^+_\mu|0\rangle$ it is convenient to evaluate the kernel between the states $|A\rangle = \varepsilon_A \cdot b^+|0\rangle$. The transition functions are $\langle A|\theta_f\rangle = \varepsilon_A\cdot\theta_f$, $\langle\theta_i^*|A\rangle = \theta_i^*\cdot\varepsilon_A$. Therefore after the integration over $\mathrm{d}\mu_f\,\mathrm{d}\mu_i$ we get

$$K_{\mathrm{PH}}(x_f, x_i)_{AB} = \varepsilon^\mu_A K_{\mathrm{PH}}(x_f, x_i)_{\mu\nu}\varepsilon^\nu_B = -\int_{-\infty}^{0} \mathrm{d}\tilde{\alpha}\int_{x_i,\tau_i}^{x_f,\tau_f} \mathscr{D}\left(x,\frac{P}{2\pi}\right)\exp\left(\mathrm{i}\int_{\tau_i}^{\tau_f}\mathrm{d}\tau\,[-P\cdot\dot{x}-\alpha(P^2-\mu^2) + \alpha(1-\lambda)P\cdot\varepsilon_A P\cdot\varepsilon_B]\right). \tag{24}$$

Here we have used the identity

$$\varepsilon^\mu_A(\mathrm{e}^F)_{\mu\nu}\varepsilon^\nu_B = \exp(\varepsilon^\mu_A F_{\mu\nu}\varepsilon^\nu_B) = (\exp(\varepsilon\cdot F\cdot\varepsilon))_{AB}. \tag{25}$$

The integrals over $P(\tau)$ and $x(\tau)$ are Gaussian and their evaluation gives the result

$$K_{\mathrm{PH}}(x_f, x_i)_{AB} = \int_{-\infty}^{0}\mathrm{d}\tilde{\alpha}\frac{\mathrm{i}}{(4\pi\tilde{\alpha})^2(\det T_{AB})^{1/2}} \times \exp[(\mathrm{i}/4\tilde{\alpha})(x_f-x_i)_\mu(T^{-1}_{AB})^{\mu\nu}(x_f-x_i)\nu + \mathrm{i}\tilde{\alpha}\mu^2] \tag{26}$$

where T^{-1}_{AB} is the inverse of the matrix

$$T^{\mu\nu}_{AB} = \eta^{\mu\nu} - \tfrac{1}{2}(1-\lambda)(\varepsilon^\mu_A\varepsilon^\nu_B + \varepsilon^\nu_A\varepsilon^\mu_B) = T^{\nu\mu}_{AB}. \tag{27}$$

Let us note that in the standard basis $\varepsilon^\mu_0 = (1,0,0,0)\ldots\varepsilon^\mu_3 = (0,0,0,1)$, $\det T_{AA}\neq 0$ for $\lambda\neq 0$, while $\det T_{AB}\neq 0$ $(A\neq B)$ for $\lambda\neq 1, 3$. Equation (26) must be evaluated at a non-singular value of λ. As the final result contains only diagonal terms, the only singular value for λ is $\lambda=0$, just the value for which there is no Green function associated with (1) for $\mu\to 0$.

By taking the Fourier transform, we get

$$\begin{aligned} K_{\mathrm{PH}}(P', P)_{AB} &= (2\pi)^{-4}\int \mathrm{d}^4x_f\,\mathrm{d}^4x_i\exp(-\mathrm{i}P'\cdot x_f + \mathrm{i}P\cdot x_i)K_{\mathrm{PH}}(x_f, x_i)_{AB} \\ &= \delta^4(P'-P)\int_{-\infty}^{0}\mathrm{d}\tilde{\alpha}\,\exp\{-\mathrm{i}\tilde{\alpha}[P^2-\mu^2-(1-\lambda)P\cdot\varepsilon_A P\cdot\varepsilon_B + \mathrm{i}\varepsilon]\} \\ &= \delta^4(P'-P)\left(\int_{-\infty}^{0}\mathrm{d}\tilde{\alpha}\,\exp\{-\mathrm{i}\tilde{\alpha}[(P^2-\mu^2)\eta-(1-\lambda)P\cdot\varepsilon P\cdot\varepsilon+\mathrm{i}\varepsilon]\}\right)_{AB} \end{aligned} \tag{28}$$

where in the last step, by using (25) and $\eta = (\eta_{AB})$, we rewrote K_{PH} as a 4×4 matrix.

We then get

$$K_{\mathrm{PH}}(P', P)_{AB} = \varepsilon^{\mu}_{A} K_{\mathrm{PH}}(P', P)_{\mu\nu} \varepsilon^{\nu}_{B} = \left(\frac{\mathrm{i}\delta^4(P'-P)}{(P^2-\mu^2)\eta - (1-\lambda)P \cdot \varepsilon P \cdot \varepsilon} \right)_{AB} \tag{29}$$

and we recover (19) with just the same calculations as starting from (18).

Therefore we have succeeded in getting a pseudoclassical description of the photon also in the covariant gauges and not only the non-manifestly covariant one in the Lorentz gauge given previously (Barducci and Lusanna 1982). As a by-product we get the pseudoclassical description of a massive spin-1 particle for $\lambda = 1$, $\mu^2 \neq 0$.

References

Barducci A and Lusanna L 1982 *The photon in pseudoclassical mechanics, University of Geneva preprint*
Faddeev L D 1976 *Les Houches, Methods in Field Theory* (Amsterdam: North-Holland)
Feldman G and Matthews P T 1962 *Massive Electrodynamics, ICTP-62-17 preprint* (unpublished)
Itzykson C and Zuber J B 1980 *Quantum Field Theory* (New York: McGraw-Hill)
Schwinger J 1951 *Phys. Rev.* **82** 664
Stückelberg E C G 1938 *Helv. Phys. Acta* **11** 225, 229